Exercise 19

Find dy/dx by implicit differentiation.

$$\sin(xy) = \cos(x + y)$$

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}[\sin(xy)] = \frac{d}{dx}[\cos(x + y)]$$

$$[\cos(xy)] \cdot \frac{d}{dx}(xy) = [-\sin(x + y)] \cdot \frac{d}{dx}(x + y)$$

$$[\cos(xy)] \cdot \left\{ \left[\frac{d}{dx}(x) \right] y + x \left[\frac{d}{dx}(y) \right] \right\} = [-\sin(x + y)] \cdot (1 + y')$$

$$[\cos(xy)] \cdot \left[(1)y + x(y') \right] = [-\sin(x + y)] \cdot (1 + y')$$

$$(y + xy') \cos(xy) = -(1 + y') \sin(x + y)$$

Solve for y'.

$$y \cos(xy) + xy' \cos(xy) = -\sin(x + y) - y' \sin(x + y)$$

$$[x \cos(xy) + \sin(x + y)]y' = -\sin(x + y) - y \cos(xy)$$

$$y' = -\frac{\sin(x + y) + y \cos(xy)}{x \cos(xy) + \sin(x + y)}$$